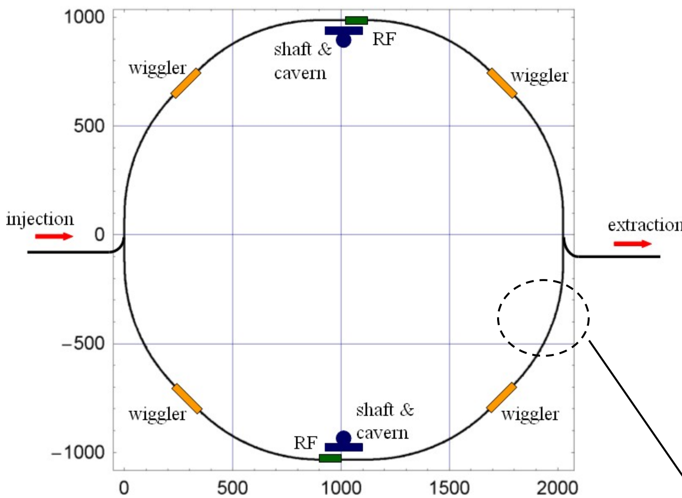
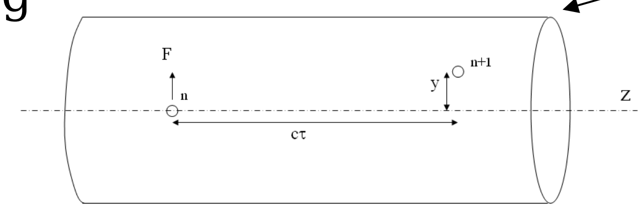
Finite wall wake function

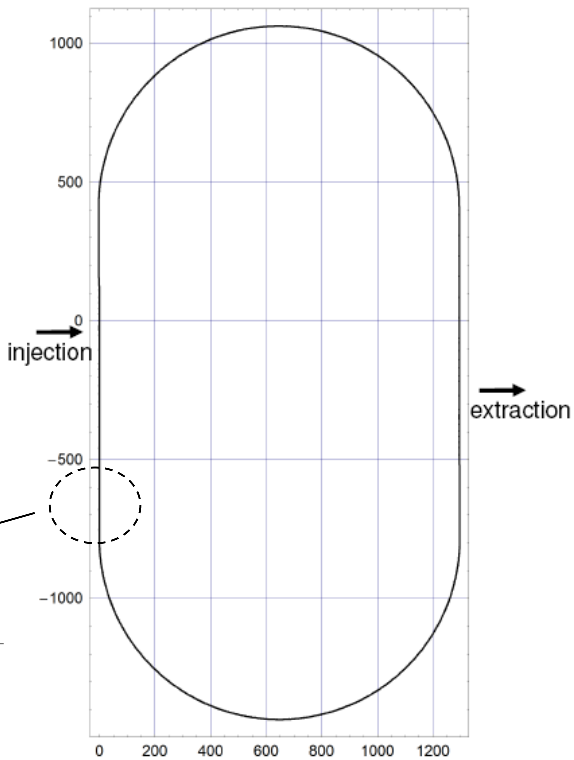
Motivation: Study of multi-bunch instability in damping rings



OCS6 damping ring



Wake field perturbs
Trailing bunches



DCO2 damping ring

Why finite wall?

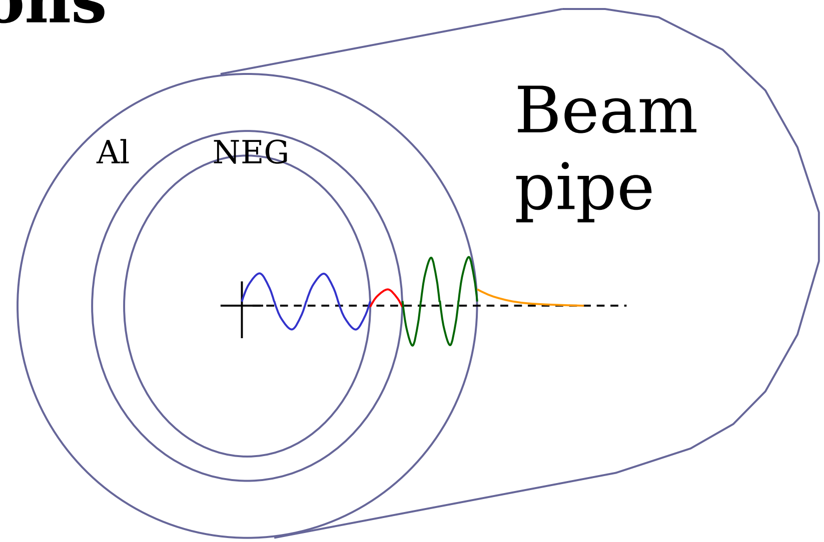
For many years (Chao 1993), we have used the approximation $\frac{1}{\sqrt{z}}$ for the resistive wall wake function. This assumes infinitely thick wall and high frequencies.

When the LHC was built, some parts of the beam pipe have wall thickness that are very thin. This motivated an interest in finite wall wake function. New solutions and formulae are published.

When I simulated the multi-bunch instability, the wake forces of many bunches must be summed before the total force converged. This required going around the damping ring 50 times.

What if the thick wall approx. is inaccurate at such large distances? Perhaps the accurate wake function decays more rapidly?

olving of Maxwell’s equations



ern (2000), Zotter (2005), Al-Khateeb (2007):

ng of boundary conditions at interfaces gives linear system of eq

conditioned because coefficients span over 12 orders of magnitu

hought that analytic solution is necessary, e.g. using *Mathematic*

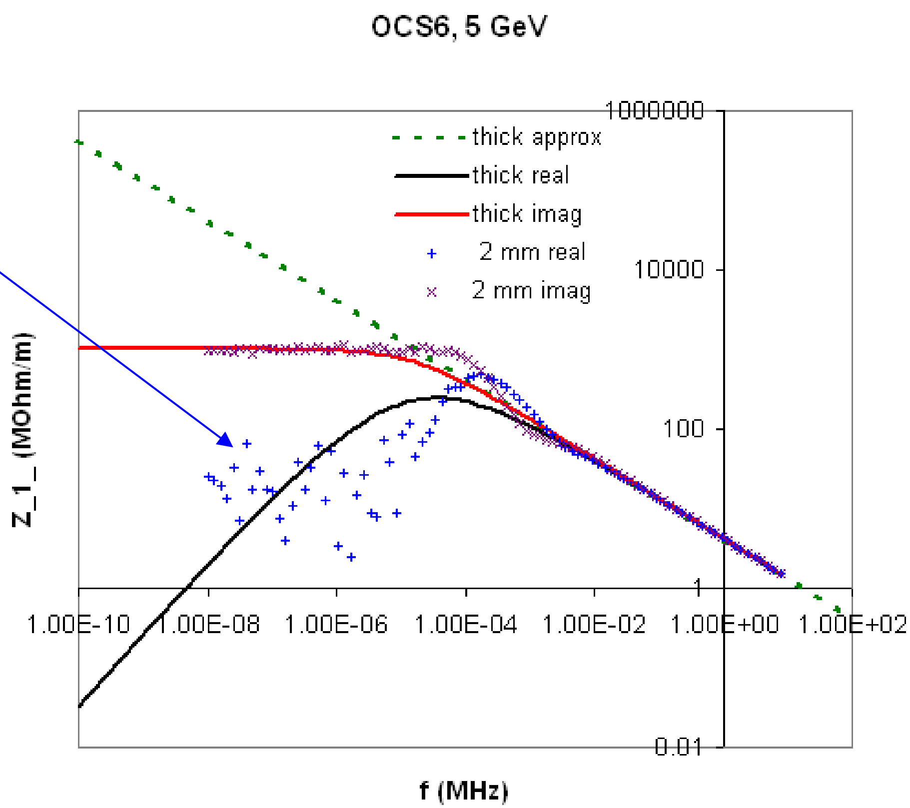
G coating is a metallic alloy used to produce better vacuum)

Problems with analytic solution

Memory requirement and symbolic calculation time increases rapidly with matrix size

In *Mathematica*, analytic calculation appears to fail when electron energy reaches 5 GeV.

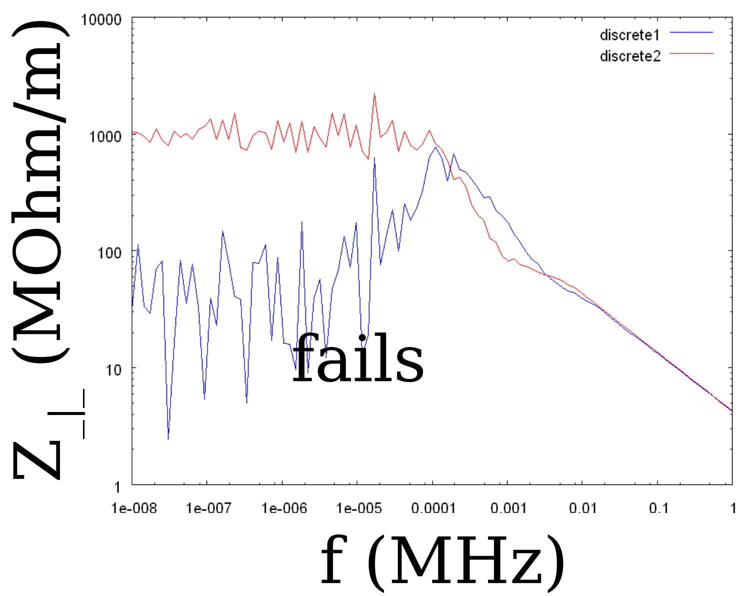
Analytic calculation fails



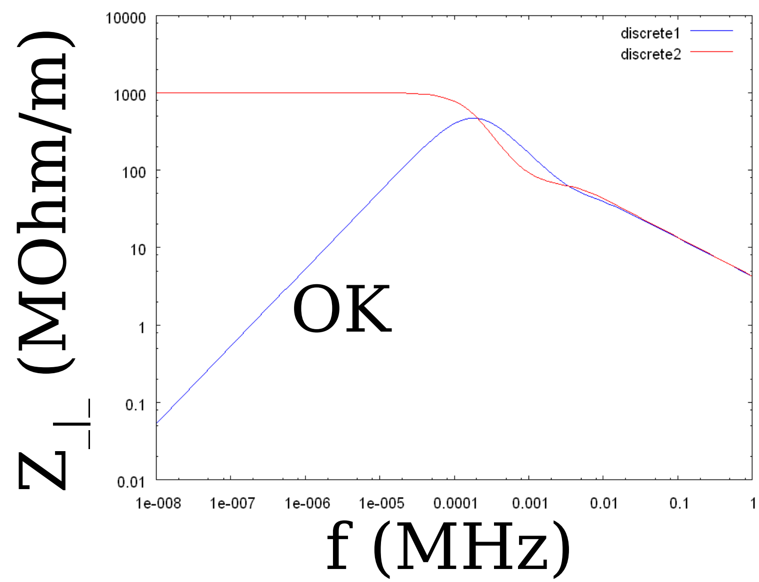
Really need Arbitrary Precision Arithmetic

Using Maxima, a free computer algebra software,
we control the floating point precision,
and apply Gaussian elimination directly.

Floating point precision = 8



Floating point precision = 16



OCS6 – 2 mm wall

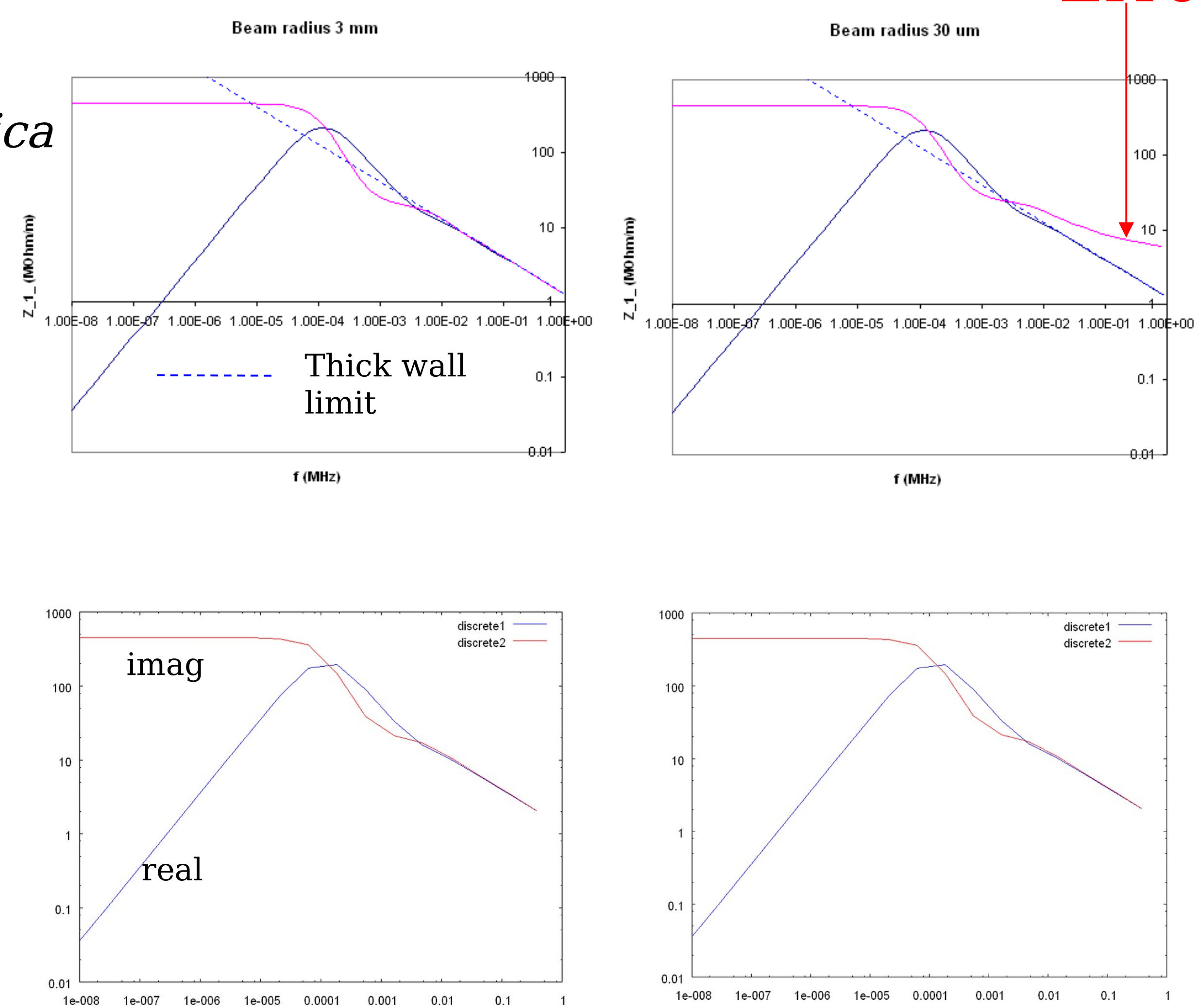
DCO2: pipe 3 cm, wall 2mm, NEG 0 um

se Gaussian elimination. No more analytic.

Error?

Mathematica

Maxima

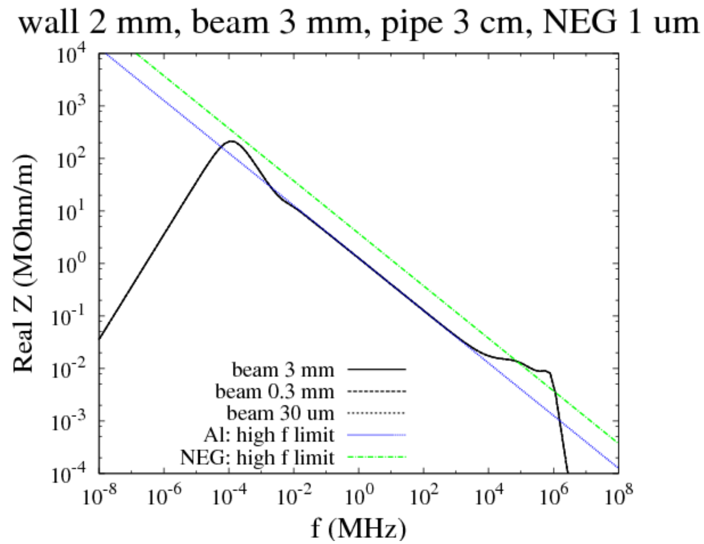


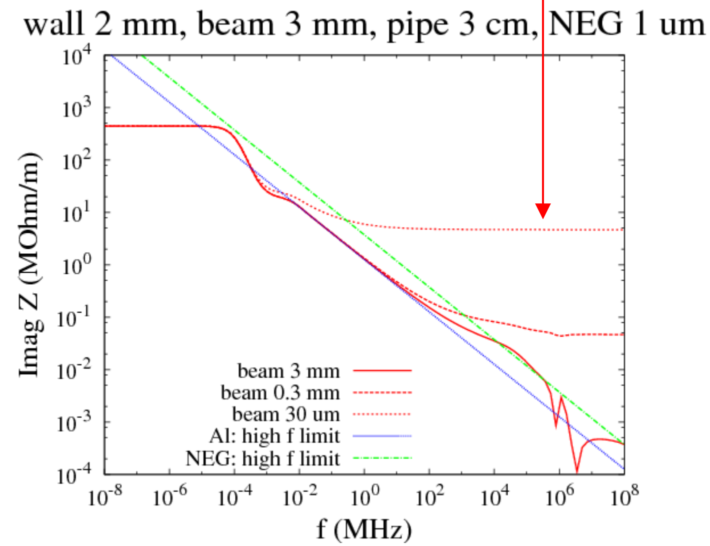
Neither gives full control of precision.
No idea which is right. Go with the majority.

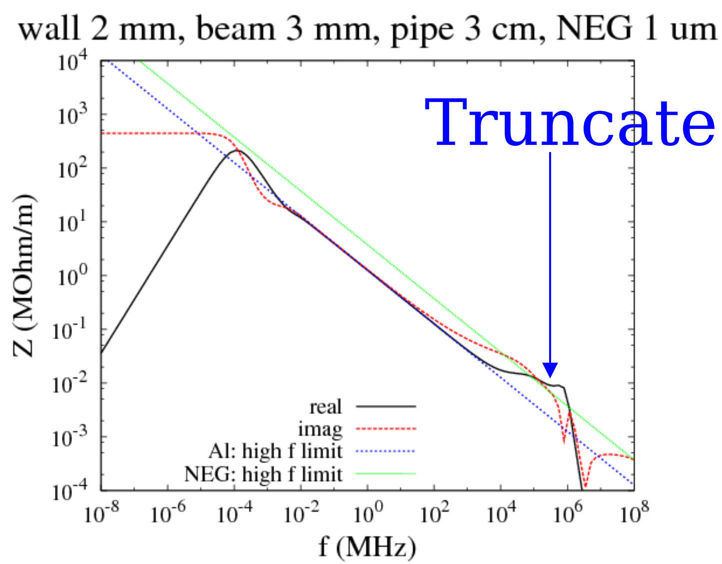
DCO2: pipe 3 cm, wall 2mm, NEG 1 um

Mathematica

Error?



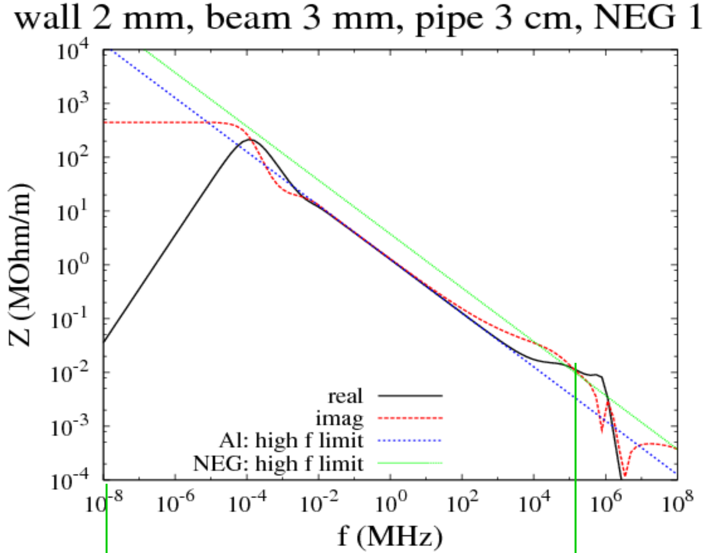


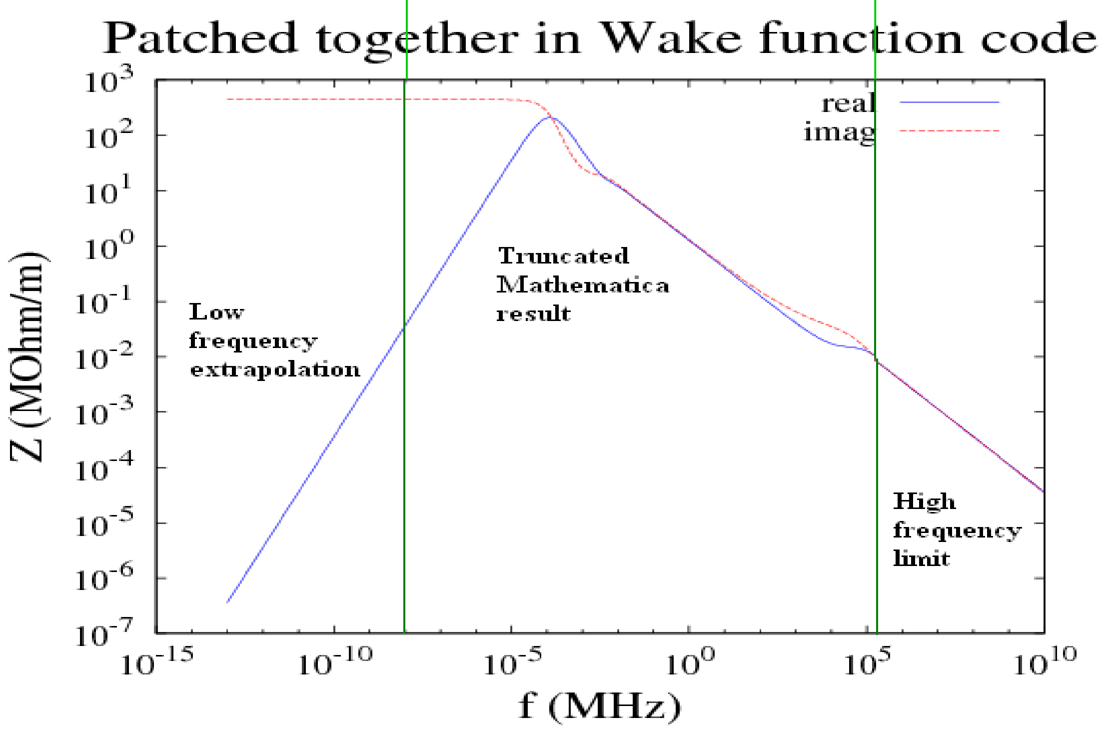


Need to write
Arbitrary Precision
Code to confirm result

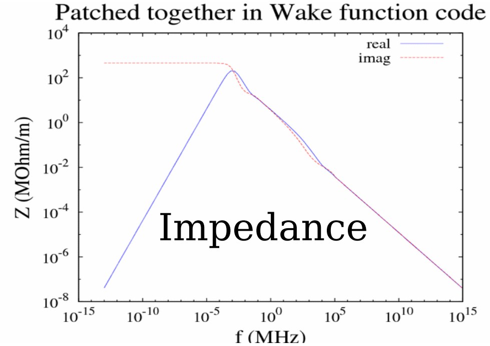
In preparation for wake function calculation

Mathematic

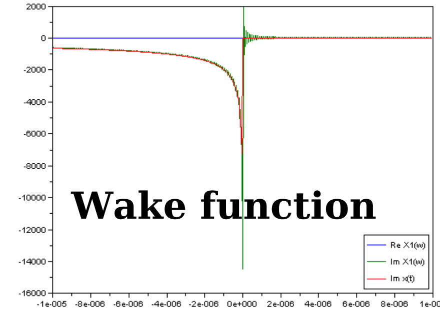




In principle:



Fourier transform →



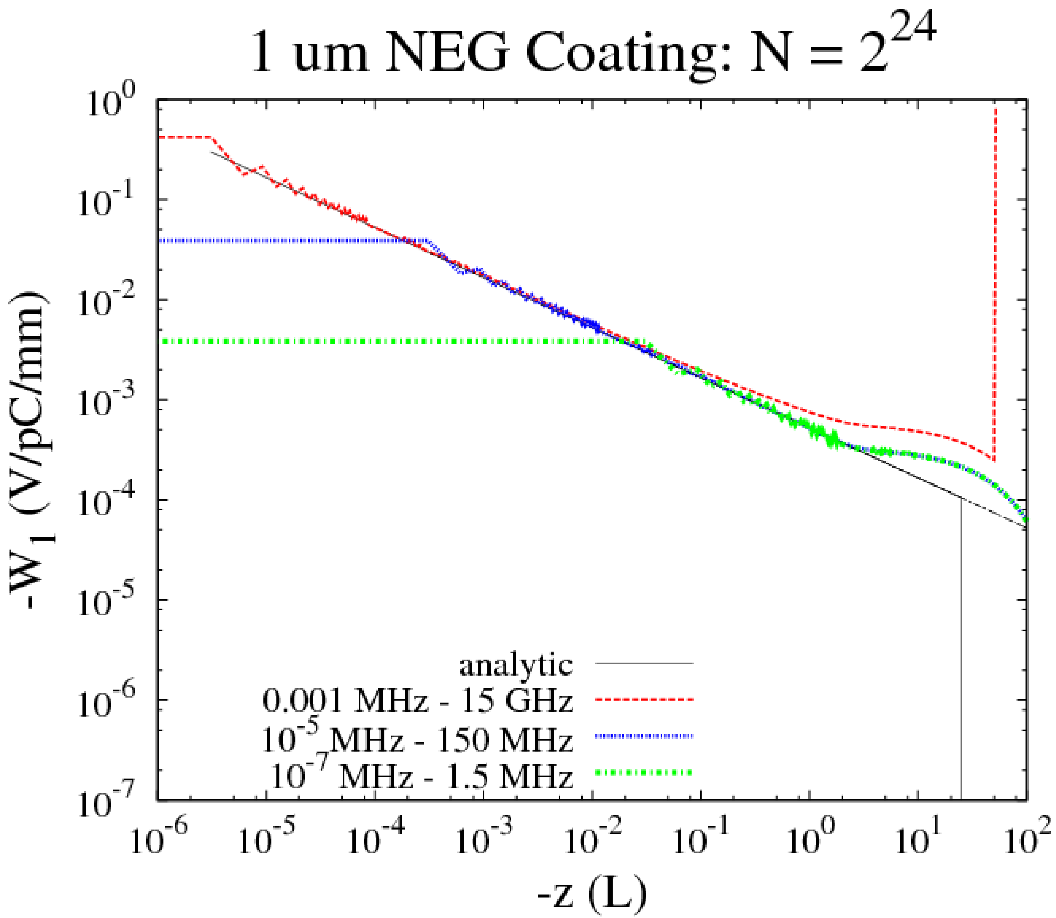
In real life …

Frequency interval must be fine enough to sample $Z(\omega)$ at low frequencies to give accurate $W_1(z)$ at large z (for wake sum convergence, about 50 x Circumference)

Maximum frequency (f_{max}) must be large enough to give accurate $W_1(z)$ at small z (for nearest bunch, about 1 m)

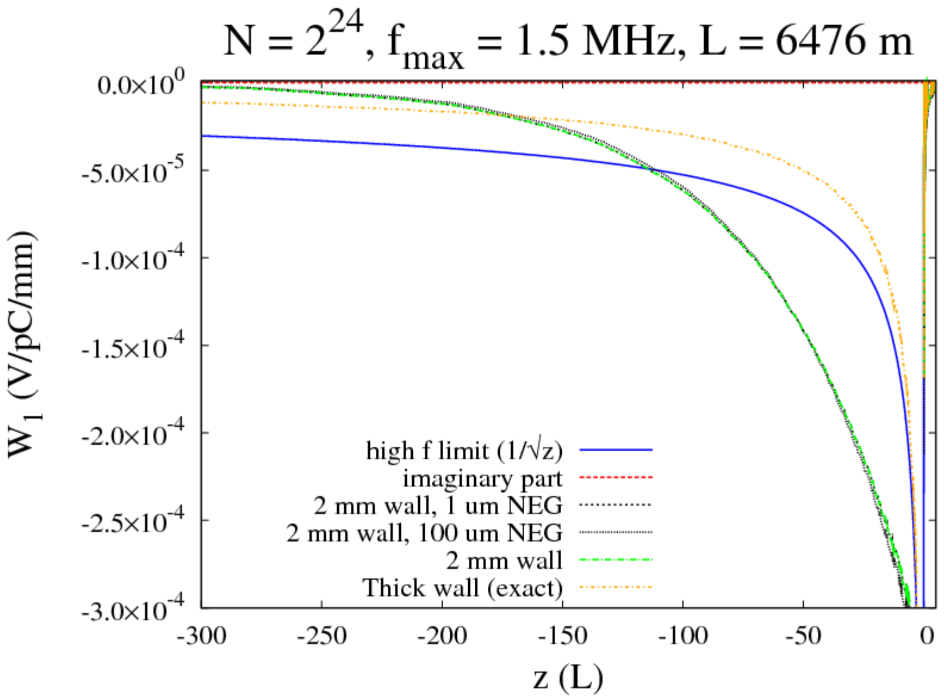
Requires: 500 GB of hard disk, lots of computer

ke Function Must be Patched Together from 3 FFT Res



Requires: less than 1 GB disk space, takes a few minutes

Wake Functions Under Different conditions



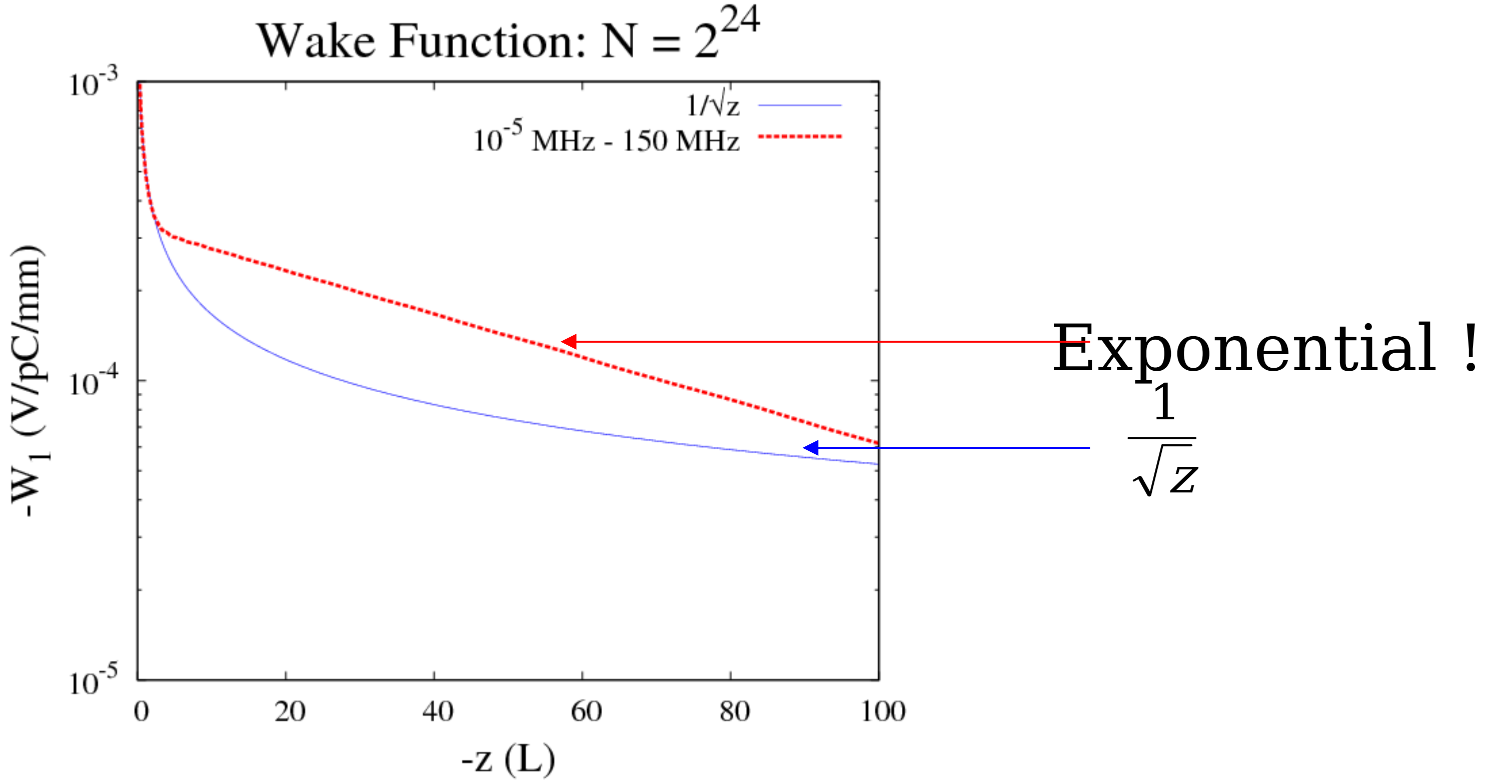
Many interesting physics here:

Actual thick wall $W_1(z)$ decays faster than $\frac{1}{\sqrt{z}}$

1 um NEG coating gives similar $W_1(z)$

2 mm wall gives higher $W_1(z)$ at small z

Vake Sum Convergence Problem Is Solved



tep ...

- Create wake function data file
- Use this in transient effect simulation
- write Arbitrary Precision code to confirm wake function r